S282 Astronomy

Wien's law

Study time: 1 hour

Summary

In this spreadsheet activity you will investigate spectra from a number of different types of star.

You will use Wien's displacement law to obtain an estimate of the surface temperatures of the stars and compare this with the temperature derived by fitting the spectrum with the Planck (black-body) function. This activity is related to work in Chapter 3 of *An Introduction to the Sun and Stars*. In particular, you should have read Sections 1.3.2 and 3.3.2 of this book before starting this activity.

Learning outcomes

- Understand Wien's displacement law.
- Appreciate the differences in stellar spectral continuum shapes.
- Handle uncertainties.
- Develop your spreadsheet skills.

Background to the activity

In this activity you will use an existing spreadsheet to explore how well stellar spectra can be modelled as black-body spectra.

Wien's displacement law

Most normal (i.e. main sequence) stars have spectra that approximate to that of a black-body source.

As described in Section 1.3.2 of *An Introduction to the Sun and Stars*, Wien's displacement law relates the temperature of a black-body source to the peak wavelength in the spectrum in a very simple way:

$$\lambda_{\text{peak}}/\text{m} = \frac{2.90 \times 10^{-3}}{T/\text{K}}$$

So an estimate of a given star's temperature can be made by measuring its spectrum and finding the peak wavelength.

The black-body spectrum: Planck's radiation law

Wien's law gives a simple relationship between temperature and peak wavelength of the spectrum. For a complete understanding of the shape of a black-body spectrum it is necessary to turn to quantum mechanics. Again, there is a law (called Planck's radiation law) that gives the shape of the spectrum for a black-body source at a given temperature. (If you are interested in the mathematical

details of this law see Appendix A – although do note that you are *not* expected to understand the mathematical details of this law.) Although *you do not need to learn this law* you will use it as part of the prepared spreadsheet to test how well a stellar spectrum compares with a true black-body spectrum. Planck's law allows us to calculate the flux density emitted from the surface of a black body.

The shape of the black-body spectrum (also known as the Planck curve) depends only on temperature and can therefore be fitted to the actual spectral data to give a more precise measurement of temperature. Note that the temperature derived in this way is based on the shape of a large part of the spectrum rather than just the peak wavelength, and since the star's spectrum only approximates a black body, the two results may be different.

The activity

The spreadsheet supplied contains data for a selection of stars of different spectral types. The spectra used in this activity have been selected from a catalogue of stellar spectra obtainable from *The Electronic Universe* at http://zebu.uoregon.edu/spectra.html

They were taken using a telescope and spectrograph at an Earth-based observatory and the spectra shown have been corrected for the variable transmission of the Earth's atmosphere. This is achieved by observing a star with a known spectrum (a 'standard star') under identical conditions, which will therefore be affected in the same way by the atmosphere. If the standard star has an observed spectrum with spectral flux density $S_{\mathrm{obs}}(\lambda)$ and a true spectrum (unaffected by the atmosphere) $S_{\mathrm{true}}(\lambda)$, then the true spectrum of any star $F_{\mathrm{true}}(\lambda)$ can be determined from its observed spectrum $F_{\mathrm{obs}}(\lambda)$ by:

$$F_{\mathrm{true}}(\lambda) = F_{\mathrm{obs}}(\lambda) \times \frac{S_{\mathrm{true}}(\lambda)}{S_{\mathrm{obs}}(\lambda)}$$

If you haven't already done so, open the spreadsheet 'Wiens_Law.sxc' from the course DVD in the usual manner.

- Start the S282 Multimedia guide program and open the 'Stars' folder, then click on the icon for this activity ('Wien's law').
- Press the Start button to access the folder on the DVD containing the StarOffice and Excel versions of the raw data file.
- Open the file you wish to use by double-clicking on it.

The spreadsheet

The spreadsheet is divided into a number of sections; below the headings are three yellow areas used for calculations.

The first of these, headed Constants, contains the values of a number of constants that will be used in the calculations.

Constants		
Speed of light	3.00E+08	m/s
Boltzmann	1.38E-23	J/K
Planck	6.63E-34	Js
nm	1.00E-09	
Wien constant	2.90E-03	Km

Although good for calculations spreadsheets tend not to be good at keeping track of units, so the units of each constant have been included explicitly in the third column. Also, note that in the spreadsheet the units have been expressed in the form 'm/s' rather than the preferred convention 'm s^{-1}', and there should also be spaces between 'Js' and 'Km' to avoid any ambiguity. ('K m' means 'kelvin metres' – do not mistake this for 'km', kilometres.)

The second area, Wien's Law, contains two white cells into which you will enter your estimates of peak wavelength and its associated uncertainty, and two yellow cells that will give the results of the Wien's law temperature calculation again with units and associated uncertainty:

Wien's Law				
Peak Wavelength		+/-		nm
Temperature		+/-		K

The actual spectral data are listed lower down, in the columns labelled Wavelength and Intensity:

Spectral Type O5					
Wavelength (nm)	Intensity		Planck fit	Scaled Planck fit	
351.0	521.2				
351.5	526.93				
352.0	528.07				

To the right of those columns are some further columns that will contain the results of the Planck curve fitting calculations.

You can select individual spectra for investigation using the tabs at the lower left-hand corner of the spreadsheet:

42	364.5	466.67			
43	365.0	465.93			

O5 / B6 / A2 / F0 / F7 / G2 / K0 / K5 / M2 / Spectra

Each sheet on the spreadsheet contains the data for a different spectral type, from O5 to M2. You can select a sheet simply by clicking on the tab. Here, the tab indicates the sheet containing the spectrum for a star of spectral type O5.

Estimate the peak wavelength

From the spectral type tabs select the sheet containing the K5 spectrum. Each sheet has the same layout with the spectral data in the Intensity column and a graph of the spectrum plotted to the right of the data columns.

By looking at the graph make an estimate of the peak wavelength. As the data are noisy (rather than a smooth curve) this will not necessarily be simply the highest data point: rather, you should try to estimate the overall maximum of the distribution. As this is not an exact process you should also make an estimate of the *uncertainty* of your value – how accurately can you judge the peak wavelength?

Record these values in column 2 of Table 1 (p. 5), and then enter them into the Wien's Law area of the spreadsheet. (Enter the wavelength, in nm, into cell G5 and the uncertainty, again in nm, into cell I5.)

Calculate the temperature

Once the wavelength has been entered the temperature is calculated using the following rearrangement of the Wien's law equation:

$$T/\mathrm{K} = \frac{2.90 \times 10^{-3}}{\lambda_{\mathrm{peak}}/\mathrm{m}}$$

The result appears in cell G8. Record this temperature and its uncertainty in column 3 of Table 1.

■ How would you convert the uncertainty in wavelength into an uncertainty in temperature?

❑ Looking at the formula the temperature is calculated as a constant divided by the wavelength. The relative uncertainty in the result will be the same as the relative uncertainty in the wavelength. Thus, if the wavelength is known to an accuracy of 10% (i.e. a relative uncertainty of 0.1) the calculated temperature will also be accurate to 10%. (Handling uncertainties was addressed in more detail in the activity 'Stellar distance and motion'.)

The sheet calculates the uncertainty in temperature in this way and places the result in cell I8.

Use the Planck radiation law to obtain best fit to a black-body spectrum

Once the estimated temperature has been calculated you can use Planck's radiation law to calculate a theoretical spectrum, which can be compared to the actual spectrum.

In the area of the spreadsheet Planck Fit enter the temperature you obtained using Wien's law. (Note that the units of 'Nm' in the spreadsheet should be 'nm'. This error shows what happens with the spreadsheet's autocorrect function – see Appendix A of the activity 'Stellar distance and motion' to learn how to switch it off.)

The spreadsheet will draw the corresponding Planck curve in blue on the same graph as the stellar spectrum. You can now adjust this temperature until you obtain what you believe to be the best overall match to the shape of the spectrum. You will not be able to get an *exact* match to the data, but you should try to get the best overall agreement you can. Note that the black-body spectrum is only scaled to place it on the same graph; it could be raised or lowered to obtain a better fit. You will need to use your judgement to estimate which temperature produces the best fit. Enter this value in column 4 of Table 1 with the corresponding peak wavelength in column 5.

Also, you can estimate the uncertainty in these values by estimating the range of temperatures over which you are able to obtain an acceptable fit. Use these values to define the uncertainty in temperature and the corresponding uncertainty in the peak wavelength, and enter the data in columns 4 and 5 of Table 1.

Write down any comments you have on your ability to perform the measurements and the agreement (or discrepancy!) between the results.

Repeat these activities for the other spectral types and complete the remaining rows in Table 1. For those at either end (O5 and M2) the peak appears to be outside of the measured wavelength range so you will only be able to make an estimate of the upper or lower limit to the temperature. However, the best fit using the Planck function may give you a better estimate. (A completed Table 1 is given 'Answers to questions' section found at the end of this activity.)

Table 1 An empty table to record your results.

1	2	3	4	5
Spectral type	**Peak wavelength measured/nm**	**Surface temperature derived from Wien's law/K**	**Surface temperature derived from fitting Planck curve/K**	**Peak wavelength obtained from fitting Planck curve/nm**
O5				
B6				
A2				
F0				
F7				
G2				
K0				
K5				
M2				

Comments

O5	
B6	
A2	
F0	
F7	
G2	
K0	
K5	
M2	

Question 1

The results you have obtained for the best fit temperature using the Planck function are unlikely to be the same as those using Wien's law and it is even possible that your estimated uncertainties may not overlap. Can you give any reason for this?

Question 2

Compare your results in Table 1 with Table 3.2 in *An Introduction to the Sun and Stars*. How well do your values for the temperatures of the different spectral types correspond with those given in the table?

Question 3

The data for the spectra in the spreadsheet contain a number of gaps, particularly towards the shorter wavelength end. What is the cause of these gaps?

Question 4

Estimating the temperature of the O and M stars is made difficult because the peaks of their spectra lie outside the visible range. Can you give two possible reasons why the spectra do not extend further into the ultraviolet or infrared?

Endnote

In this activity you have attempted to determine stellar temperatures by applying Wien's law and Planck's law to real stellar spectra. You will have recognized that although the shape of stellar spectra approximate to black-body curves there are significant differences at some wavelengths, which can lead to erroneous estimates of temperature. In particular, you will have recognized that your estimates of uncertainties based on the accuracy of your measurement of a quantity do not always provide a reliable estimate of the true uncertainty. This is because you are making an implicit assumption when using those measurements (i.e. that the stellar spectrum is a black body) which is not true. Temperatures derived by studying the relative strengths of spectral lines will often be more reliable (see Section 3.3.2 in *An Introduction to the Sun and Stars*).

Answers to questions

Question 1

You would only expect to get the same answer if the star's spectrum was *the same* as a black-body and there were no observational errors.

Planck function fitting is the best technique since the whole of the spectrum is used rather than just one part, i.e. the peak.

Comment

For the B6, A2 and F0 stars the difficulty with fitting the Planck function arose because the spectrum appears to be too low (compared with the Planck function) at short wavelengths (i.e. less than ~400 nm). In this part of the spectrum the hydrogen Balmer lines (see Section 3.3.2 of *An Introduction to the Sun and Stars*) bunch closer and closer together, corresponding to transitions of electrons from energy level $n = 2$ to higher and higher levels. These transitions result from absorption of radiation by the stars' atmospheres (see Section 1.3.2 in *An Introduction to the Sun and Stars*). At wavelengths shorter than 365 nm photons can ionize the hydrogen and so are relatively easily absorbed, causing a dip in the spectrum called the Balmer discontinuity. Other significant deviations from

black-body spectra occur when molecules are present, such as TiO where the combination of many lines forms bands in the spectra of type M stars.

Question 2

Table 2 shows and example of the results that you might obtain from performing this activity. You may find that your answers are in some cases rather different from those in Table 3.2 in *An Introduction to the Sun and Stars* even allowing for the uncertainties in your results. This is likely to be due to the difference between the true spectral shapes of the stars and the black-body curves used for comparison (see comment above). This absorption can also affect the position of the peak of the spectrum giving an erroneous solution from Wien's law (e.g. for the B6 star).

Table 2 Example of the results obtained by performing this activity.

1	2	3	4	5	
Spectral type	**Peak wavelength measured/nm**	**Surface temperature derived from Wien's law/K**	**Surface temperature derived from fitting Planck curve/K**	**Peak wavelength obtained from fitting Planck curve/nm**	**Temperature derived from Table 3.2 in *The Sun and Stars*/K**
O5	< 350	> 8300	45 000 ± 5 000	64 ± 6	40 000
B6	380 ± 10	7600 ± 200	14 000 ± 1 000	205 ± 20	15 000
A2	400 ± 10	7250 ± 200	9500 ± 500	305 ± 20	9500
F0	405 ± 10	7200 ± 200	7500 ± 500	390 ± 30	7400
F7	440 ± 20	6600 ± 300	6600 ± 300	440 ± 20	6500
G2	450 ± 10	6450 ± 150	6100 ± 300	475 ± 25	5800
K0	480 ± 20	6050 ± 250	5400 ± 200	540 ± 20	4900
K5	610 ± 20	4750 ± 150	4400 ± 200	660 ± 30	4100
M2	> 800	< 3600	3200 ± 300	910 ± 90	3200

Comments

O5	Upper limit to peak wavelength gives lower limit to temperature
B6	Large discrepancy between T derived from peak wavelength and Planck curve fit. Hard to fit Planck curve – large amount of absorption at short wavelength end of spectrum
A2	T from peak wavelength lower than from Planck fit. Absorption at short wavelength end of spectrum prevents fitting Planck curve well
F0	Impossible to fit Planck curve well so large error results
F7	Hard to fit Planck curve
G2	None
K0	Large discrepancy between T obtained from Wien's law and Planck fit
K5	Good Planck function fit obtained
M2	Hard to tell if peak has been reached within observed spectrum. Lower limit to peak wavelength gives upper limit to temperature

Question 3

The observations were made with a ground-based telescope and spectrometer. The atmosphere absorbs radiation at certain wavelengths (see Figure 1.38 in *An Introduction to the Sun and Stars*) and the gaps probably occur where the atmospheric transmission is not sufficiently high to provide good data.

Question 4

(i) The detector may not be sensitive to radiation at these wavelengths. (This is likely to be the case for the infrared end of the spectrum.)

(ii) The atmosphere may absorb the radiation. (This is likely to be the case at the ultraviolet end of the spectrum.)

Resources

The Electronic Universe http://zebu.uoregon.edu/spectra.html

Appendix A The black-body spectrum

Planck's law describes the shape of a black-body spectrum and allows us to calculate the spectral flux density emitted by a black-body surface using:

$$F_\lambda/(\mathrm{W\,m^{-2}\,nm^{-1}}) = \frac{2\pi hc^2}{\lambda^5}(e^{hc/\lambda kT} - 1)^{-1}\,\Delta\lambda \quad \text{where } \Delta\lambda \text{ is } 10^{-9}\text{ m}$$

In this equation, λ is the wavelength, h is the Planck constant (6.63×10^{-34} J s), c is the speed of light (3.00×10^{8} m s^{-1}), k is the Boltzmann constant (1.38×10^{-23} J K^{-1}) and T is the temperature. e is a number (2.718…), which is raised to the power $hc/\lambda kT$ in this equation.